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Flutter Boundary Prediction Based on Nonstationary Data Measurement

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A time-domain method for the flutter boundary prediction of a wing based on the nonstationary random response is proposed. The feature of this method is that a stability parameter is used as the measure of the aeroelastic stability margin instead of a conventional modal damping. The effectiveness of a stability parameter for the flutter boundary prediction is shown using a simple bending–torsion wing model. Numerical studies are conducted to examine the performance of the method under the nonstationary condition. An application to supersonic wind-tunnel flutter testing data shows that the proposed method is effective to the prediction of the flutter boundary in the actual flutter testing.

Nomenclature

$a_1 - a_n$	=	autoregressive coefficients
$b_1 - b_m$	=	moving average coefficients
$E[\ ]$	=	expectation
$e(k)$	=	white noise
$F^-(n-1)$	=	stability parameter
I	=	unit matrix
m	=	order of moving average polynomial
n	=	order of autoregressive polynomial
p	=	number of modes included in sampled measurements
q	=	dynamic pressure
q_F	=	flutter boundary dynamic pressure
T	=	sampling interval
$v(k)$	=	measurement noise
$y(k)$	=	sampled wing response
z^{-1}	=	backward shift operator, $z^{-1}y(k) = y(k-1)$
η_i	=	damping coefficient of ith mode
$\boldsymbol{\theta}(k)$	=	autoregressive moving average coefficients vector, $\{a_1, \ldots, a_n, b_1, \ldots, b_m\}^T$
ρ	=	forgetting factor
$\sigma_y, \sigma_e, \sigma_v$	=	standard deviation of $y(k)$, $e(k)$, and $v(k)$
ω_i	=	angular frequency of ith mode

Superscript

T	=	transpose of a matrix

Introduction

SINCE the onset of flutter gives restriction on the flight envelope, the determination of the flutter boundary is one of the most important subjects in structural dynamics of airplane design and development. According to the flutter analysis, wind-tunnel and flight-flutter testing are conducted at a final stage. Flutter testing is generally conducted at the subcritical condition to avoid structural damage during testing. Therefore, an accurate prediction of the flutter boundary is necessary based on the available information, and so flutter boundary prediction techniques have been developed.[1–11]

The flutter boundary is generally predicted from the behavior of a stability criterion against the flight speed or dynamic pressure. The most usual criterion to measure the stability of the dynamical system is damping of the unstable mode, and a significant amount of research has focused on the accurate estimation of damping.[1–5] However, it is pointed out that damping is not always a suitable measure to predict the boundary. The deficiency of damping as a flutter stability indicator is that in the case of a so-called explosive flutter it starts to decrease suddenly near the critical point. Zimmermann and Weissenburger[6] gave their attention to this problem and proposed a new stability criterion called the flutter margin, which is based on Routh's stability criteria. They showed that the flutter margin was expressed by the parabolic form of the dynamic pressure from the analysis of a two-dimensional wing model. Though the original stability margin was defined only for a two-degree-of-freedom system, an extension to a three-degree-of-freedom system was done by Price and Lee.[7] However, the flutter margin is not suitable for use with recently developed system identification techniques since it is evaluated in the continuous time system. As an appropriate method to use with the time series analysis or system identification procedures, Matsuzaki and Ando[8] proposed the stability parameter based on Jury's stability criteria[12] for a sampled-data system, and showed that the decrease of the stability parameter was more gradual than damping. The effectiveness of the stability parameter as the estimation index of the flutter boundary has been shown in wind-tunnel flutter testing[8,9] and in numerical studies.[10,11]

Wind-tunnel and flight-flutter testing are very time consuming because the stationary tests are repeated under several conditions. Also, real-time identification of flutter stability is important from the point of safety during flutter testing. Therefore, real-time evaluation of system stability or an accurate and reliable flutter boundary prediction applicable to nonstationary flutter testing has been required. Walker and Gupta[2] presented the near-real-time flutter analysis (NRTFA) program based on the recursive prediction error (RPE) method, where dampings were calculated from the autoregressive coefficients estimated. To improve the estimate of damping, Roy and Walker[3] used the extended Kalman filter as a real-time parameter identifier.

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Attempts to obtain a better estimation of damping under nonstationary conditions have been continued.[4] On the other hand, Matsuzaki and Ando[9] used the scattering property of the stability parameters estimated in short data blocks to predict the flutter boundary from nonstationary testing data. The effectiveness of the method was confirmed in both experiments[9] and numerical studies.[10,11] However, this approach was not essentially a nonstationary method because the stationary time series analysis was used to identify the aeroelastic system.

The objective of the present study is to propose a new time-discrete flutter boundary prediction method based on nonstationary random response and to examine its effectiveness. In the method, wing response is adaptively identified with the autoregressive moving average (ARMA) process, and the dynamical stability of the system is evaluated using a stability parameter calculated from the autoregressive (AR) coefficients identified at each sampling step. As the real-time estimator of ARMA parameters, the recursive maximum likelihood (RML) algorithm was used. First, numerical studies using a simple bending–torsion wing are carried out to investigate the property of the stability parameter estimated under nonstationary conditions. Then the result of application to the wind-tunnel flutter testing data is shown, in which dynamic pressure is increased at a constant rate, and compared with the dampings estimated from the same data.

Theoretical Background

Stability Evaluation of Aeroelastic System

Since a wing is always excited randomly by turbulence of an airstream, the sampled response $y(k)$ are considered to be a random time series. Then the system is represented by an ARMA model:

$$A(z^{-1})y(k) = B(z^{-1})e(k) \tag{1}$$

where $A(z^{-1})$ and $B(z^{-1})$ are defined by

$$A(z^{-1}) = 1 + a_1 z^{-1} + \cdots + a_n z^{-n} \tag{2}$$

$$B(z^{-1}) = 1 + b_1 z^{-1} + \cdots + b_m z^{-m} \tag{3}$$

The order of AR polynomial is set to twice the number of modes included in sampled measurements:

$$n = 2p$$

and the MA order m is determined by Akaike's Information Theoretic Criterion (AIC), if a preliminary study is possible, or set to $n - 1$ in case of no prior information.

The characteristic polynomial of the system expressed in Eq. (1) is

$$G(z) = z^n + a_1 z^{n-1} + \cdots + a_n \tag{4}$$

Here, the Jury's stability criteria,[12] a stability test for the discrete system, is applied to evaluate the system stability. It is shown that among the check parameters, $F^-(n - 1)$ gives a critical dynamic pressure in the flutter boundary prediction problem, that is, it has a positive value at a subcritical condition and gets to zero at the flutter boundary.[13] We call it a stability parameter and use it as the indicator of an aeroelastic stability margin instead of a critical modal damping. The definition of $F^-(n - 1)$ is

$$F^-(n - 1) = \det(X_{n-1} - Y_{n-1}) \tag{5}$$

where

$$X_{n-1} = \begin{bmatrix} 1 & a_1 & \cdots & a_{n-2} \\ 0 & 1 & \ddots & \vdots \\ \vdots & \ddots & \ddots & a_1 \\ 0 & \cdots & 0 & 1 \end{bmatrix}$$

$$Y_{n-1} = \begin{bmatrix} a_2 & \cdots & a_{n-1} & a_n \\ \vdots & \iddots & a_n & 0 \\ a_{n-1} & \iddots & \iddots & \vdots \\ a_n & 0 & 0 & 0 \end{bmatrix}$$

The flutter boundary is predicted by extrapolating a fitting curve to $F^-(n - 1)$ plotted against the dynamic pressure.

Parameter Estimation of ARMA Model

The ARMA coefficients a_i and b_i are updated at every sampling instant according to the RML algorithm given by[14]

$$\boldsymbol{\theta}(k) = \boldsymbol{\theta}(k - 1) + \boldsymbol{L}(k)\varepsilon(k) \tag{6}$$

$$\varepsilon(k) = y(k) - \boldsymbol{\varphi}^T(k)\cdot\boldsymbol{\theta}(k - 1) \tag{7}$$

$$\boldsymbol{L}(k) = \frac{P(k - 1)\boldsymbol{\psi}(k)}{\rho + \boldsymbol{\psi}^T(k)P(k - 1)\boldsymbol{\psi}(k)} \tag{8}$$

$$P(k) = \frac{1}{\rho}\left[P(k - 1) - \frac{P(k - 1)\boldsymbol{\psi}(k)\boldsymbol{\psi}^T(k)P(k - 1)}{\rho + \boldsymbol{\psi}^T(k)P(k - 1)\boldsymbol{\psi}(k)}\right] \tag{9}$$

where

$$\boldsymbol{\psi}^T(k) = \{-\tilde{y}(k - 1), \ldots, -\tilde{y}(k - n), \tilde{\varepsilon}(k - 1), \ldots, \tilde{\varepsilon}(k - m)\}$$

$$\boldsymbol{\varphi}^T(k) = \{-y(k - 1), \ldots, -y(k - n), \hat{\varepsilon}(k - 1), \ldots, \hat{\varepsilon}(k - m)\}$$

and $\tilde{\varepsilon}(k)$, $\hat{\varepsilon}(k)$, and $\tilde{y}(k)$ are given by

$$\hat{\varepsilon}(k) = y(k) - \boldsymbol{\varphi}^T(k)\cdot\boldsymbol{\theta}(k) \tag{10}$$

$$\tilde{\varepsilon}(k) = \hat{\varepsilon}(k) - b_1(k - 1)\tilde{\varepsilon}(k - 1) - \cdots - b_m(k - 1)\tilde{\varepsilon}(k - m) \tag{11}$$

$$\tilde{y}(t) = y(t) - b_1(t - 1)\tilde{y}(t - 1) - \cdots - b_m(t - 1)\tilde{y}(t - m) \tag{12}$$

To start the algorithm, the initial values $\boldsymbol{\theta}(0)$ and $P(0)$ are necessary. If no prior information is available, a common choice is

$$\boldsymbol{\theta}(0) = 0, \qquad P(0) = \alpha \cdot I \tag{13}$$

where α is a scalar number. The larger α becomes, the more quickly the estimate of parameters converges to the true values.

In parameter identification under the nonstationary condition, the tracking performance of the algorithm is very important. The ability to follow the characteristic changes is adjusted by ρ. If ρ is set to less than 1.0 then the estimate shows a quick tracking, whereas it has a larger variation as the drawback. Therefore, an appropriate choice of ρ according to the test condition is necessary to obtain the reliable prediction of the flutter boundary.

Numerical Simulation

Quick convergence to the true values and tracking to the change of the system dynamics, as well as accurate estimation, are the important abilities of the nonstationary method. In this section the behavior of the stability parameter estimated under several nonstationary conditions are investigated using numerical simulation. Then the prediction performance of the flutter boundary is shown, and the robustness to measurement noise is tested.

Wing Model

Figure 1 shows a two-dimensional bending–torsion wing used in this study. Unsteady incompressible flow theory is applied to derive an aerodynamic pressure load. The parameters used in this study are the same as in Ref. 10; $\omega_h = 50$ rad/s, $\omega_\alpha = 100$ rad/s, $a = -0.4$, $x_\alpha = 0.2$, $r_\alpha^2 = 0.25$, and $\mu = 40$. The resulting equations of motion can be expressed in the time-discrete state-space form:

$$\boldsymbol{x}(k+1) = A\boldsymbol{x}(k) + \boldsymbol{B}e(k) \tag{14}$$

$$y(k) = \boldsymbol{Cx}(k) + v(k) \tag{15}$$

where

$$E[e(i)e(j)] = \sigma_e^2 \cdot \delta_{ij}, \quad E[v(i)v(j)] = \sigma_v^2 \cdot \delta_{ij}, \quad E[e(i)v(j)] = 0$$

The system is excited by a random sequence $e(k)$, and the generated $y(k)$ is contaminated by $v(k)$. Data are generated at $T = 0.02$ s.

The eigenvalue analysis of Eq. (14) gives the theoretical values of $F^-(3)$, ω_i, and η_i. Figure 2 shows the comparison between $F^-(3)$ and η_1 plotted against the normalized dynamic pressure q/q_F. While η_1 is increased up to $q/q_F = 0.8$, $F^-(3)$ starts to decrease at around $q/q_F = 0.5$ and is approximated quite well by a quadratic curve. This figure demonstrates that a stability parameter is a better criteria as the flutter predictor than a damping parameter in this example.

Initial Convergence

At the starting period of the recursive parameter estimation, it is an important requirement for the real-time method that the estimated parameter converges to the true value from the initial condition as soon as possible. In the RML algorithm, ρ and the initial value $P(0)$ $(=\alpha I)$ have great influence on the quick convergence. To illustrate the improvement of a convergence speed, the transient behavior of the estimated stability parameter is observed for several values of ρ and α. Since the data include two vibrational modes, ARMA(4,3), which means that the order of AR and MA polynomials are 4 and 3, is chosen as the identification model. The initial ARMA coefficients $\boldsymbol{\theta}(0)$ is set to zero, which means that $F^-(3)$ starts from 1. The same stationary data at $q/q_F = 0.6$ with no measurement noise is used for all tests.

For a rapid convergence, ρ should be smaller and α should be larger. The estimation results for three cases are presented in Fig. 3. This figure shows that adjusting both parameters improves the speed of settling down to the true value. For $\rho = 0.99$ and $\alpha = 10^5$, the estimation of $F^-(3)$ gets to the goal within 4 s, which corresponds to 200 samplings. These are thought to be reasonable iteration counts.

Tracking Ability

The tracking performance to the change of the system characteristics is a primary concern in nonstationary data analysis. A common technique to improve the tracking ability is the use of the appropriate value of ρ for the test condition. To test the tracking performance, the stability parameter method is applied to nonstationary data whose dynamic pressure is increased at a constant rate from $q/q_F = 0.6$–1.

As case 1, the increasing rate of the dynamic pressure is set to $\mathrm{d}q/\mathrm{d}t = 0.004q_F$. It takes 100 s for this test case, which corresponds to 5000 sampling data. Figure 4 illustrates the effect of ρ under condition 1. If the forgetting factor is not used, that is, $\rho = 1.0$, the tracking ability is very poor. Introducing ρ of less than 1 improves the tracking performance significantly. A smaller value of ρ, however, makes the estimate sensitive to the local characteristics of the random response, so that the estimated parameter has a larger fluctuation.

To examine the availability of use in a more rapid increasing rate, two more nonstationary conditions were tested, that is, the increasing rate is set to $\mathrm{d}q/\mathrm{d}t = 0.008q_F$ in case 2 and $\mathrm{d}q/\mathrm{d}t = 0.02q_F$ in case 3. For case 2 the parameter was estimated using $\rho = 0.996$ and depicted in Fig. 5. Figure 6 is the esti-

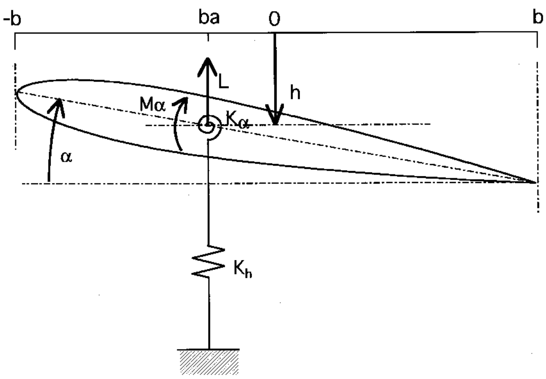

Fig. 1 Two-dimensional wing section.

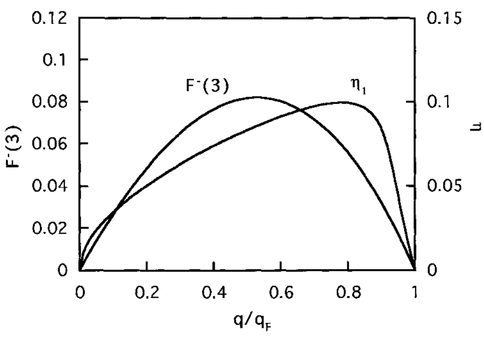

Fig. 2 Comparison of $F^-(3)$ and η_1.

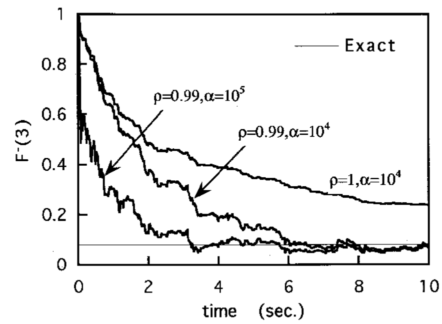

Fig. 3 Effect of ρ and α to the initial parameter convergence.

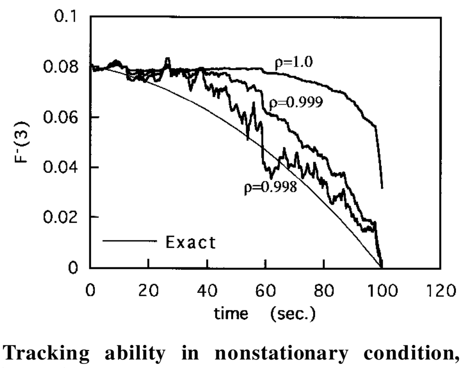

Fig. 4 Tracking ability in nonstationary condition, $\mathrm{d}q/\mathrm{d}t = 0.004q_F$ (case 1).

mation result using ρ = 0.993 for case 3. These results show that the estimated parameter will follow the system variation if the value of ρ is chosen appropriately.

Flutter Prediction

Generally, flutter testing is conducted in the subcritical condition, so that the flutter boundary has to be predicted from the available information. To predict the critical dynamic pressure, the stability parameters plotted against the dynamic pressure are approximated by a fitting curve and then the curve is extrapolated to the horizontal axis. Since the stability parameter of the simulation model is approximated very well by a quadratic function of the dynamic pressure, as shown in Fig. 2, a quadratic curve was used here for the flutter prediction.

Figure 7 shows the prediction errors obtained from the estimate using ρ = 0.998 in case 1, where the flutter prediction was done in the range starting from $q/q_F = 0.6$ to the several dynamic pressures denoted on the horizontal axis; for example, if the data of $q/q_F = 0.6$–0.8 are used to predict the flutter, the error is about 6% and for $q/q_F = 0.6$–0.95 is 0.5%, etc. If data are measured up to $q/q_F = 0.85$ or 0.9, the flutter boundary is predicted within 6 or 3% error.

To compare with a conventional damping method, ω_i and η_i were evaluated from the same data using the same RML algorithm.

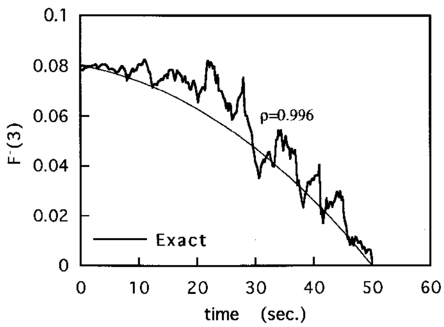

Fig. 5 Tracking ability in nonstationary condition, $dq/dt = 0.008q_F$ (case 2).

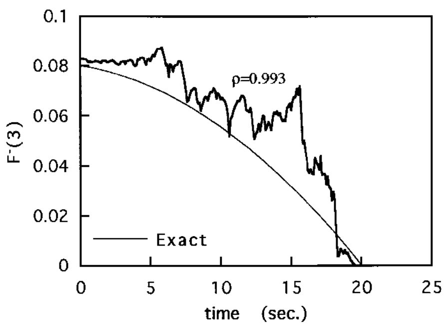

Fig. 6 Tracking ability in nonstationary condition, $dq/dt = 0.02q_F$ (case 3).

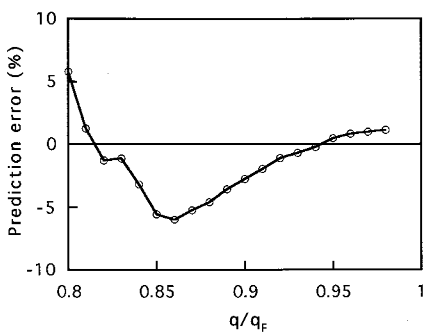

Fig. 7 Prediction error of flutter boundary of a two-dimensional wing model.

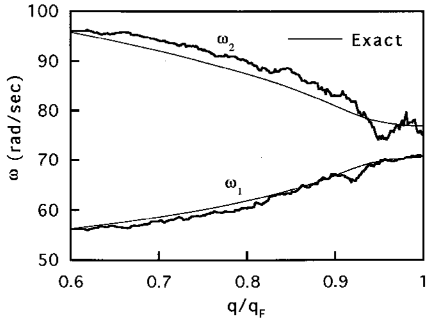

Fig. 8 Estimation results of ω_1 and ω_2.

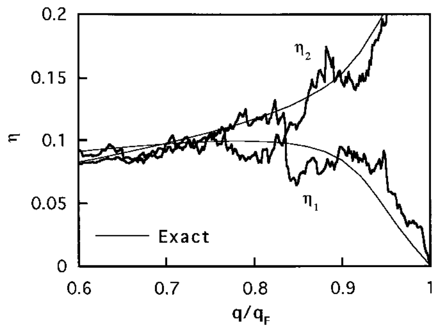

Fig. 9 Estimation results of η_1 and η_2.

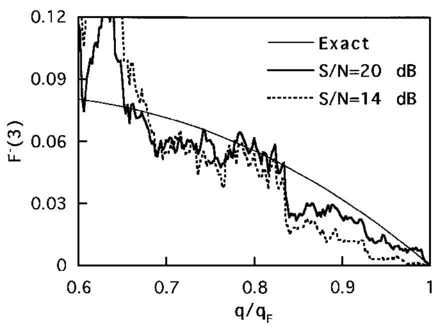

Fig. 10 Effect of measurement noise to the estimation of stability parameter $F^-(3)$.

gorithm. The estimations of ω_i are fairly accurate and steady, as shown in Fig. 8. Figure 9 indicates that the estimated η_i track the variation well, though they fluctuate more than ω_i around the true value. However, since the critical damping itself starts to decrease around $q/q_F = 0.85$, it is impossible to predict the flutter boundary effectively using the data up to $q/q_F = 0.85$ or 0.9, unlike the stability parameter.

Robustness to Noise

To assure the robustness to measurement noise, data contaminated with a white noise are used to estimate the stability parameter. The test condition is the same as case 1. The signal-to-noise ratio used are $S/N = 14$ and 20 dB. Here S/N is defined as

$$S/N = 20 \log_{10}(\sigma_y/\sigma_v), \text{ dB}$$

The results are illustrated in Fig. 10. Although the larger noise causes a magnification of variance, the method still works properly for these noise levels.

Application to Wind-Tunnel Flutter Testing

To assess the effectiveness of the stability parameter method to the actual test data analysis, the data record of the supersonic wind-tunnel flutter testing conducted in the National

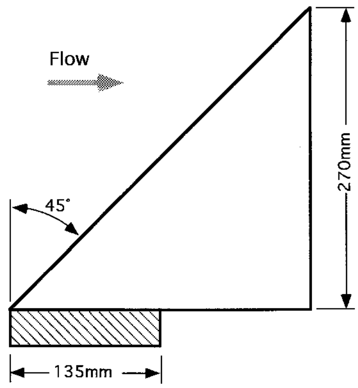

Fig. 11 Planform of a delta wing.

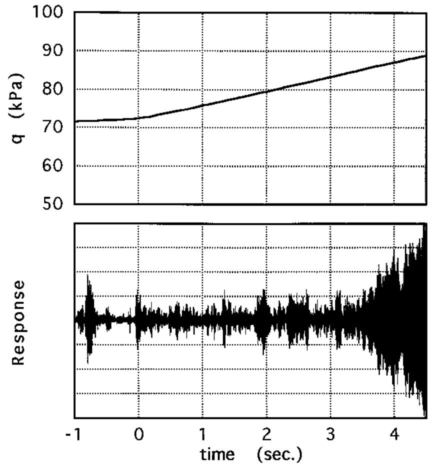

Fig. 12 Time history of dynamic pressure and wing response.

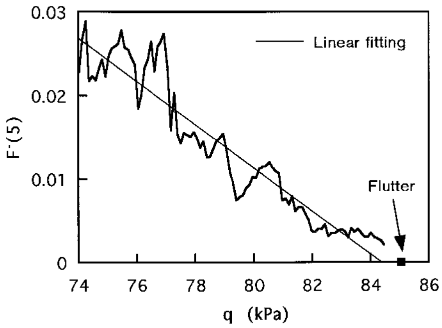

Fig. 13 Estimation results of stability parameter $F^-(5)$ using ρ = 0.995.

Aerospace Laboratory was employed. The planform of the test wing is depicted in Fig. 11. It is made of aluminum alloy of 1.75 mm thickness and has a double-wedge of 2 mm width at the leading and trailing edges. Only a half-chord from the leading edge of the wing root is mounted. This configuration is designed so that flutter occurs by coupling of the second and third mode within a testing dynamic pressure range. The Mach number was fixed at 2.5 and the dynamic pressure was increased at a constant rate. The dynamic pressure of the wind tunnel and the time history of sampled wing response against time are shown in Fig. 12. The flutter boundary observed in the experiment was q = 84.9 kPa. An analog bandpass filter of bandwidth 60–180 Hz was used to cut higher modes and a low-frequency noise. The filtered signal was digitized by a 12 bit-length A/D converter at sampling interval T = 0.003 s.

Since the data involve three vibration modes, ARMA(6,2) was used as the identification model, where the order of the

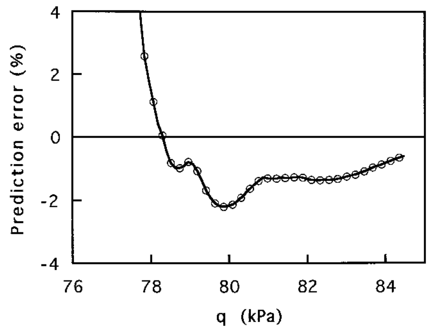

Fig. 14 Prediction error of flutter boundary of a delta wing.

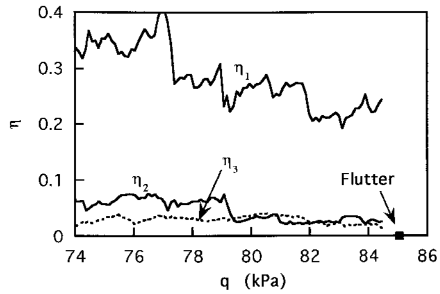

Fig. 15 Estimation results of η_1–η_3.

MA part was determined using the AIC. Estimation was made for q = 72.0–84.5 kPa (for about 3.3 s), and ρ was set to 0.995. Figure 13 shows that the estimated $F^-(5)$ decreases monotonically toward zero, so that the linear fitting gives an accurate and reliable flutter prediction. The prediction error of the boundary dynamic pressure is plotted in Fig. 14 in the same manner as in Fig. 7. This figure shows that the prediction error is within about 2% if the maximum dynamic pressure of the test is more than q = 78 kPa.

Dampings evaluated from the same data are plotted in Fig. 15. This figure shows that monitoring the unstable modal damping (the second mode) does not give an accurate prediction of the flutter onset for this testing.

Conclusions

The flutter boundary prediction method based on nonstationary data measurement was proposed. The advantage of the stability parameter used here as the indicator of the aeroelastic system stability was shown in the analysis of a two-dimensional wing model in comparison with the unstable damping coefficient. Numerical simulations were conducted to investigate the estimation performance under nonstationary conditions and the effectiveness of the stability parameter method. An application to supersonic wind-tunnel flutter testing showed that the proposed method worked successfully in the actual experimental data processing and gave an accurate and reliable prediction of the flutter boundary dynamic pressure. However, dampings evaluated from the same estimation of ARMA parameters did not give any useful information about the flutter boundary for this testing.

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